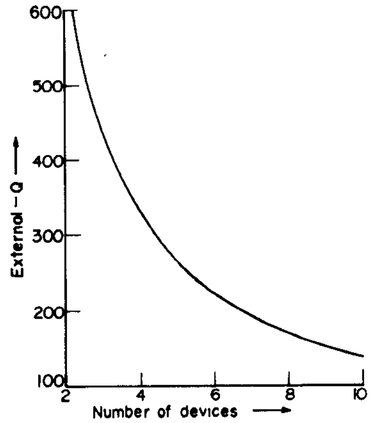

Fig. 2. External-Q as a function of the number of devices.

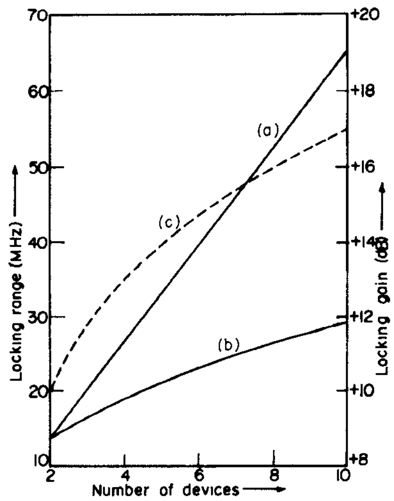

Fig. 3. Functions of the number of devices. (a) Locking range with constant gain. (b) Locking range with constant injection power. (c) Locking gain with constant injection power.

devices is changed. From (4), (5), and (7), it can be easily seen that

$$\Delta f_r = \frac{F_D g_o}{\pi C_D \sqrt{A}} \left(1 + \frac{C_c}{n^2 N C_D}\right)^{-1} \tag{9}$$

when the locking gain is kept constant by proportionately increasing P_t as the devices are increased in number.

Fig. 3 shows the variation of the locking range with an increase in the number of devices for a multiple-device oscillator of $F_D = 1.41$, $v_o = 18.26$ V, $C_D = 0.3$ pF, $C_c = 10$ pF, $n = 0.4$, $g_o = 3$m mho, and $G_c = 0.01$m mho. Two cases are shown. One obtained from (9) for a constant locking gain of 9.96 dB, which is the locking gain of the oscillator for $N = 2$ and $P_t = 100$ mW. For the other case, which is a plot of (8), injection power is constant at 100 mW and the locking gain increases with an increase in the number of devices (broken line in Fig. 3) along with a decrease of Q_{ext} (Fig. 2). The two cases thus illustrated show that, in general, the locking range of a multiple-device oscillator increases as its constituent devices are increased in number. A comparison of the two cases indicates that, with the gain constant at a low level, a larger deviation in the locking range is obtained as the devices are increased in number. In this case, full advantage of the fall in Q_{ext} with an increase in the number of devices is taken. When injection power is constant, as the devices are increased in number, the favorable effect of the corresponding decrease in Q_{ext} is diminished by the accompanying increase in the locking gain. As a consequence, the deviation in the locking range with an increase in the number of devices is not as large as it is in the constant gain case. Thus, when the constituent devices of a multiple-device oscillator are increased in number under a constant locking-gain condition, the resulting rise in the locking range is determined by an increase in the function Q_{ext}^{-1}. When the number of devices is increased, with the primary objective of increasing the locking gain, the dependence of the locking range on the number of devices is determined by the function $(Q_{ext}A^{\frac{1}{2}})^{-1}$.

IV. Conclusion

The analysis just presented shows that the Q_{ext} of a multiple-device oscillator falls towards a limiting value as the number of devices is increased. This limiting value happens to be the quality factor of an individual device. The decrease of Q_{ext} with an increase in the number of devices can be fully utilized for increasing the injection-locking range if the locking gain is kept constant at the minimum possible level. Such a measure, of course, will lead to an increasing demand on the injection power as the devices are increased in number. In many applications, the devices are increased in number to meet high-gain requirements. In such cases, the improvement of the locking range with an increase in the number of devices is comparatively less than what may be achieved with the locking gain remaining constant at the minimum possible level.

References

[1] K. J. Russel, "Microwave power combining techniques," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 472–478, May 1979.
[2] K. Chang and C. Sun, "Millimeter-wave power combining techniques," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 91–107, Feb. 1983.
[3] K. Kurokawa, "The single-cavity multiple-device oscillators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 793–801, Oct. 1971.
[4] R. Aston, "Techniques for increasing the bandwidth of a TM_{010} mode power combiner," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 479–482, May 1979.
[5] Y.-E.Ma and Cheng-Sun, "1-W millimeter-wave Gunn diode combiner," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 1460–1463, Dec. 1980.
[6] S. Nogi and K. Fukui, "Locking behavior of a microwave multiple-device ladder oscillator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 253–262, Mar. 1985.

Invariant Definitions of the Unloaded Q Factor

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***Abstract*—Equations are presented for computing the unloaded Q factor of a microwave resonator embedded in an impedance-transforming lossless, reciprocal two-port. Knowledge of the transformation properties of the two-port is not required.**

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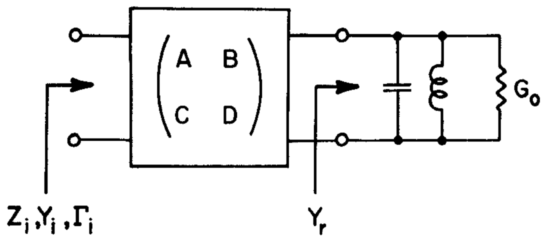

Fig. 1. Measurement of the resonator through an impedance transforming two-port.

The experimental evaluation of the Q factor of a typical microwave resonator usually necessitates observing the resonator through an unknown two-port, as shown in Fig. 1. In this figure, the resonator is represented by a parallel resonant circuit, the admittance of which is given approximately by

$$Y_r = G_0 + j\frac{dB_r}{d\omega}(\omega - \omega_0). \tag{1}$$

As a function of frequency, the locus of Y_r is a vertical straight line which crosses the real axis at G_0. This occurs at resonant frequency ω_0. The rate of change of susceptance with frequency $dB_r/d\omega$ is nearly constant in the vicinity of resonance.

The unloaded Q factor of the resonator, in terms of circuit elements, is given by [1]

$$Q_0 = \frac{\omega_0}{2G_0}\frac{dB_r}{d\omega}. \tag{2}$$

From the observation of admittance Y_r as a function of frequency, it would be possible to determine the quantities that are needed in the right-hand side of (2) in order to compute Q_0. Unfortunately, this admittance cannot be observed experimentally because Y_r is transformed into an input impedance Z_i

$$Z_i = \frac{BY_r + A}{DY_r + C}. \tag{3}$$

The complex numbers A to D are the elements of the chain matrix for the impedance-transforming two-port. Impedance Z_i is accessible to the measuring device, typically a network analyzer.

Whereas the locus of the resonator admittance is a straight line on the complex plane, the transformed impedance Z_i (as well as its inverse Y_i) describes a circle on the complex plane [2]. The position and size of the circle depend on the constants A to D. These constants are usually unknown. However, in comparison with the rapid frequency variation of Y_r (typical unloaded Q factor values are between 1000 and 10 000), it is justified to assume that A to D are independent of frequency. Furthermore, the impedance transforming two-port is assumed to be lossless and reciprocal.

Solving (3) for Y_r we have

$$Y_r = \frac{-CZ_i + A}{DZ_i - B}. \tag{4}$$

Using (4), it is possible to determine the quantities needed for the evaluation of Q_0. First, the real part of Y_r is found to be

$$G_0 = \frac{\mathrm{Re}(Z_i)}{|DZ_i - B|^2}. \tag{5}$$

Next, the absolute value of the derivative of the admittance Y_r is computed, noting that only the imaginary part of Y_r depends on frequency

$$\left|\frac{dY_r}{d\omega}\right| = \frac{dB_r}{d\omega} = \frac{\left|\frac{dZ_i}{d\omega}\right|}{|DZ_i - B|^2}. \tag{6}$$

Substituting (5) and (6) into (2) one obtains

$$Q_0 = \frac{\omega_0\left|\frac{dZ_i}{d\omega}\right|}{2\mathrm{Re}(Z_i)}. \tag{7}$$

This equation makes it possible to express the unloaded Q factor in terms of impedance Z_i, measured at the input side of the impedance transforming two-port.

Similar derivation shows that the unloaded Q factor in Fig. 1 can also be expressed in terms of the input admittance $Y_i = 1/Z_i$ as follows:

$$Q_0 = \frac{\omega_0\left|\frac{dY_i}{d\omega}\right|}{2\mathrm{Re}(Y_i)}. \tag{8}$$

Finally, in terms of the input reflection coefficient Γ_i, the unloaded Q factor can be expressed as

$$Q_0 = \frac{\omega_0\left|\frac{d\Gamma_i}{d\omega}\right|}{1 - |\Gamma_i|^2}. \tag{9}$$

Equations (7)–(9) have an invariant behavior in the sense that they are valid for the resonator embedded in an arbitrary two-port. The remarkable property of these equations is that knowledge of the transformation constants A to D is not needed in order to determine the unloaded Q factor. The only requirements on the impedance transforming two-port are, first, to be insensitive to small frequency variations and, second, to be lossless and reciprocal.

Another convenient property of the above invariant expressions is their relative insensitivity to the value of the observation frequency. In other words, the value of ω_0, the unloaded resonant frequency, does not have to be known very accurately. When the input reflection coefficient $\Gamma_i(\omega)$ is observed, it is easy to identify the loaded resonant frequency, i.e., the frequency at which $|\Gamma_i|$ is minimum. This loaded resonant frequency is slightly different from the unloaded resonant frequency [3], but for high values of Q_0, the difference is negligible when substituted in (7), (8), or (9). Also, the derivatives appearing in these equations, after being divided by the appropriate denominators, yield results which are independent of frequency.

We found these invariant expressions to be very useful in measurement of the unloaded Q factor with the aid of an automated network analyzer. The derivatives of the measured quantities have been evaluated numerically by the use of finite differences, and the results were found to be consistent.

References

[1] R. Beringer, "Resonant cavities as microwave circuit elements," in *Principles of Microwave Circuits*, C. G. Montgomery, R. H. Dicke, and E. M. Purcell, Eds. New York: Dover, 1965, ch. 7.

[2] B. A. Galwas, "Scattering matrix description of microwave resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 669–671, Aug. 1983.

[3] D. Kajfez, "Correction for measured resonant frequency of unloaded cavity," *Electron. Lett.*, vol. 20, pp. 81–82, Jan. 19, 1984.